The Measurement of Complex Reflection Coefficient by Means of a Five-Port Reflectometer

SHIHE LI, MEMBER, IEEE, AND RENATO G. BOSISIO, MEMBER, IEEE

Abstract —A detailed analysis of the five-port reflectometer is presented in this paper. The possibility and application limitation of using a five-port to measure complex reflection coefficients is discussed and the related design consideration is derived. From the point of view of minimum measurement error, an optimized, multi-octave five-port circuit is suggested and realized by ordinary microwave integrated circuit technology. Some measurement results are reported to prove the above analysis and design consideration.

I. Introduction

During the development period of the six-port technique, the possibility of using a relatively simple five-port junction to measure a complex reflection coefficient had been discussed by Engen [1], [2], and Woods [3], and some experimental five-port reflectometers have been published [4]–[7]. All of the above literature has reached the same conclusion, namely that a) the application of a five-port is limited by an ambiguity which comes from the sign of a square root in the five-port solution, and b) it is possible to measure a passive network ($|\Gamma| \leqslant 1$) by the use of an appropriately designed five-port reflectometer [2], [4], [6].

Practically, most applications of a reflectometer are related to measurements on passive networks. Therefore, a suitable five-port can meet most of the needs of automatic analysis on a one-port microwave network.

Generally speaking, the five-port is an arbitrary linear network (Fig. 1), as is the six-port. For arriving at higher measurement accuracy, practical five-ports can be constructed in the following two ways.

1) *Five-Port R (FPR)*: It was suggested by Riblet [4], [5], and is composed of one directional and two omni-directional couplers (Fig. 2(a)). The solution of Γ of DUT is the intersection of two circles which have fixed origins in the Γ plane as shown in Fig. 2(b).

2) *Five-Port L (FPL)*: It was first reported by the authors [6], and then by Martin *et al.* [7]. As a special example, the FPL reported in the above papers is a three-probe system (Fig. 3(a)), and the solution of an unknown reflection coefficient is found by two circles, but their origins vary with the Γ under test (Fig. 3(b)). It will be shown that the advantage of FPL is higher measurement accuracy in the case of similar calibration and power measurement errors.

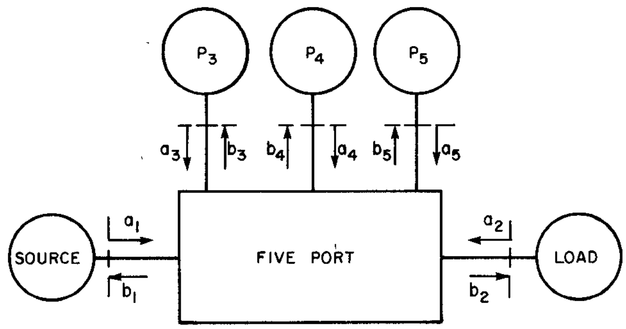

Fig. 1. The five-port network.

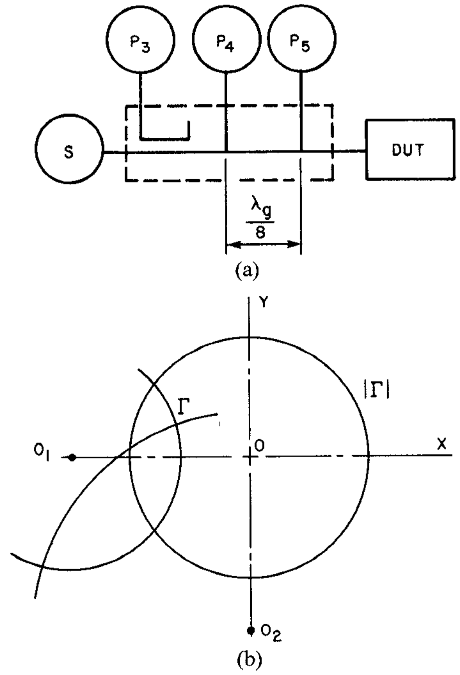

Fig. 2. The five-port R reflectometer. (a) Basic circuit of FPR. (b) Measurement principle of FPR.

In this paper, the general theory, the ambiguity problem, and design consideration of the five-port reflectometer will be discussed. Optimized FPL design is suggested, and some experimental results are reported.

Manuscript received July 14, 1982; revised December 2, 1982.
The authors are with the Electrical Engineering Department, Ecole Polytechnique de Montreal, Montreal, Quebec, Canada H3C 3A7.

0018-9480/83/0400-0321$01.00

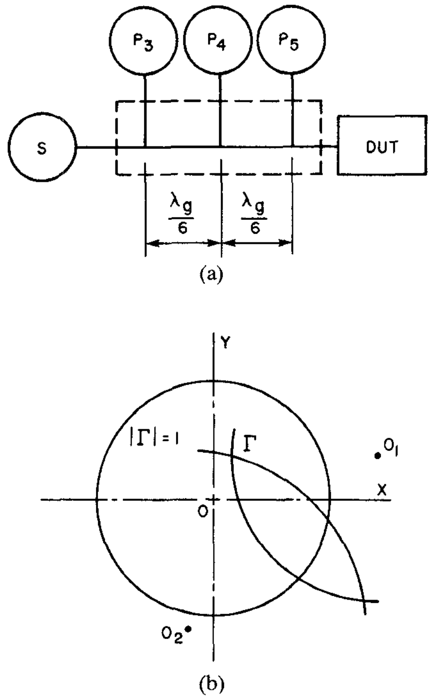

Fig. 3. The five-port L reflectometer. (a) Basic circuit of FPL. (b) Measurement principle of FPL.

II. Analysis of the Five-Port Reflectometer

According to the basic concept of the five-port [1], the three power readings from an arbitrary linear five-port junction (Fig. 1) can be written as

$$p_1 = \frac{P_4}{P_3} = q_1 \left| \frac{1 + A_1\Gamma}{1 + A_0\Gamma} \right|^2 \quad (1)$$

$$p_2 = \frac{P_5}{P_3} = q_2 \left| \frac{1 + A_2\Gamma}{1 + A_0\Gamma} \right|^2 \quad (2)$$

where P_3, P_4, and P_5 are three power readings from port 3, 4, and 5, respectively; q_1 and q_2 are two scalar coefficients; and A_i are three complex network parameters defined as follows:

$$A_i = \alpha_i e^{j\phi_i} = c_i + jd_i$$
$$i = 0, 1, \text{and } 2. \quad (3)$$

The complex reflection coefficient of DUT is

$$\Gamma = |\Gamma| e^{j\psi} = x + jy. \quad (4)$$

For convenience, (1) and (2) can be rewritten as

$$x^2 + y^2 + 2u_i x + 2v_i y = 2r_i \quad (5)$$

where

$$u_i = (p_i c_0 - q_i c_i)/\omega_i \quad (6a)$$
$$v_i = (q_i d_i - p_i d_0)/\omega_i \quad (6b)$$
$$r_i = (q_i - p_i)/2\omega_i \quad (6c)$$
$$\omega_i = p_i \alpha_0^2 - q_i \alpha_i^2. \quad (6d)$$

A. Five-Port Solution

The unknown reflection coefficient can be expressed (from (5) and (6)) as follows:

$$y = S_1 \pm \sqrt{S_1^2 - S_2} \quad (7)$$

$$x = \frac{r_1 - r_2 + (v_1 - v_2)y}{u_1 - u_2} \quad (8)$$

and

$$|\Gamma| = \frac{2}{u_1 - u_2}\left[(v_1 u_2 - v_2 u_1)y + r_2 u_1 - r_1 u_2\right] \quad (9)$$

where

$$S_1 = \frac{(r_1 - r_2)(v_1 - v_2) + (u_1 - u_2)(u_2 v_1 - v_2 u_1)}{(u_1 - u_2)^2 + (v_1 - v_2)^2} \quad (10a)$$

$$S_2 = \frac{(r_1 - r_2)^2 + 2(u_1 - u_2)(u_2 r_1 - r_2 u_1)}{(u_1 - u_2)^2 + (v_1 - v_2)^2}. \quad (10b)$$

Obviously, this solution is the intersection of two circles in the Γ plane. In special cases, the above solution can be simplified. For example, if $\omega_i = 0$, then the ith equation of (5) becomes a straight line; if $u_1 = u_2$ (or $v_1 = v_2$), the two circles will be symmetric to axis x (or y) of Γ plane [8].

B. Ambiguity

The ambiguity problem of the five-port arises from (7), because it does not give any information about the sign of the square root. This is a very important problem which will limit the application of the five-port. To describe this problem more clearly, the five-port can be simulated by the following procedure. If a load has its known reflection coefficient $\Gamma_k = |\Gamma_k| e^{j\psi_k}$ under test, then the solution (9) can be rewritten as

$$|\Gamma_m|^2 = |\Gamma_k|^2 + 2(1 \pm 1) \cdot \frac{(Z_1 + |\Gamma_k|^2 Z_0)\left[Z_1 - |\Gamma_k|^2 Z_0 + |\Gamma_k|(Z_3 \cos\psi_k + Z_2 \sin\psi_k)\right]}{(Z_3 - 2|\Gamma_k| Z_0 \cos\psi_k)^2 + (Z_2 - 2|\Gamma_k| Z_0 \sin\psi_k)^2} \quad (11)$$

where $|\Gamma_m|$ is the amplitude of measured reflection coefficient

$$Z_0 = \alpha_0\alpha_1\alpha_2\left[\alpha_0 \sin(\phi_1 - \phi_2) + \alpha_1 \sin(\phi_2 - \phi_0) + \alpha_2 \sin(\phi_0 - \phi_1)\right] \quad (12a)$$

$$Z_1 = \alpha_1\alpha_2 \sin(\phi_1 - \phi_2) + \alpha_2\alpha_0 \sin(\phi_2 - \phi_0) + \alpha_0\alpha_1 \sin(\phi_0 - \phi_1) \quad (12b)$$

$$Z_2 = \alpha_0(\alpha_1^2 - \alpha_2^2)\cos\phi_0 + \alpha_1(\alpha_2^2 - \alpha_0^2)\cos\phi_1 + \alpha_2(\alpha_0^2 + \alpha_1^2)\cos\phi_2 \quad (12c)$$

$$Z_3 = \alpha_0(\alpha_1^2 - \alpha_2^2)\sin\phi_0 + \alpha_1(\alpha_2^2 - \alpha_0^2)\sin\phi_1 + \alpha_2(\alpha_0^2 - \alpha_1^2)\sin\phi_2. \quad (12d)$$

All the network parameters α_0, α_1, α_2, ϕ_0, ϕ_1, and ϕ_2 are defined in (3). It is obvious that one of the roots of (11) is the true reflection coefficient $|\Gamma_k|$, and the other is a false root. If $|\Gamma_k|$ is kept constant but ψ_k varied from 0 to 2π, then the loci of true solutions of $|\Gamma_k|$ are a group of concentric circles, but the modulus of the false roots would vary with ψ_k when $|\Gamma_k| \neq 0$. This makes it possible to find the true reflection coefficient. For example, two methods have been suggested in the literature [8] to find the true solution without any limitation on the reflection coefficient under test. But all these possible methods are inconvenient for practical application because the main advantages of the five-port simplicity would be lost. Therefore, an acceptable measurement range of a five-port could be limited to applications for which $|\Gamma| \leqslant 1$. Under this limitation, one way is to design a five-port such that only one root of (9) or (11) is located in the unit circle of Γ plane and the other one outside of the unit circle when a passive network is under test (as suggested by Engen [2]). The other consideration is to design the five-port such that the amplitude of the true solution is always less than that of the false one. Then the smaller value of $|\Gamma|$ in (9) or (11) can be taken. From (11), it means the second term must be positive, or the following equation must be satisfied:

$$(Z_1 + Z_0)\left[Z_1 - Z_0 \pm \sqrt{Z_2^2 + Z_3^2}\right] > 0 \tag{13}$$

where the phase ψ_k has taken the worst case.

As an example, the two types of five-port are discussed as follows.

FPR: An ideal FPR has the network parameters

$$A_0 = 0 \quad A_1 = \alpha \quad \text{and} \quad A_2 = \alpha e^{j\phi} \tag{14}$$

where α is the amplitude of A_1 and A_2, and ϕ is the phase difference between A_2 and A_2.

Substituting (14) into (12) and (13)

$$\alpha < \cos\frac{\phi}{2}. \tag{15}$$

In Riblet's paper, $\phi = \pi/2$ and $\alpha = 1$, only small reflections can be measured (the published value is $|\Gamma| \leqslant 1/3$); for measuring any passive network, α must be decreased to 1/3 by adding an attenuator between the five-port and DUT [4]. Theoretically, (15) shows the value of α must be less than 0.7 when $\phi = \pi/2$, if a FPR is to be used in any passive load measurement.

FPL: The network parameters of an ideal FPL are

$$A_0 = \alpha \quad A_1 = \alpha e^{j\phi} \quad \text{and} \quad A_2 = \alpha e^{j2\phi}. \tag{16}$$

Obviously, the principle schematic (Fig. 3, when the coupling between main transmission line and the three probes are weak enough) shows a special example of $\alpha = 1$ and $\phi = 2/3\pi$. Substitute (16) into (13), the condition of measuring any passive network becomes

$$\alpha < 1. \tag{17}$$

These are the basic considerations to avoid ambiguity in a five-port design.

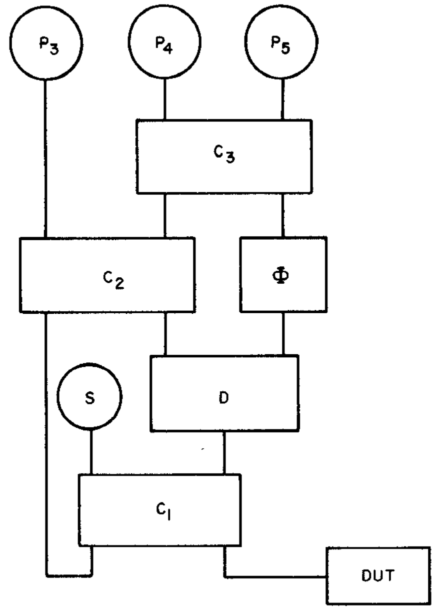

Fig. 4. Principle circuit of a multi-octave and optimized FPL reflectometer, where D is the in phase power divider; Φ is the phase shifter; and C_1, C_2, and C_3 are the three 90° symmetrical directional couplers (hybrid junction) with their respective coupling coefficients of c_1, c_2, and c_3.

III. Five-Port Design

A wide-band FPR circuit had been reported by Riblet [5]. Here, a multi-octave FPL circuit is suggested as shown in Fig. 4. In the ideal case, the network parameters of this circuit can be easily found as follows:

$$q_1 \propto \frac{c_2^2(1-c_3^2)}{1-c_2^2} \tag{18}$$

$$q_2 \propto \frac{c_2^2 c_3^2}{1-c_2^2} \tag{19}$$

$$A_0 = \frac{0.707 c_1 c_2}{\sqrt{1-c_2^2}} \exp(j90°) \tag{20}$$

$$A_1 = \frac{0.707 c_1 \sqrt{1 - c_2^2 + c_2^2 c_3^2 + 2c_3\sqrt{(1-c_2^2)(1-c_3^2)}\,\sin\theta}}{c_2\sqrt{1-c_3^2}} \cdot \exp\left\{-j\,\mathrm{tg}^{-1}\left[\mathrm{tg}\,\theta + \frac{\sqrt{(1-c_2^2)(1-c_3^2)}}{c_3\cos\theta}\right]\right\} \tag{21}$$

$$A_2 = \frac{0.707 c_1}{c_2 c_3}\sqrt{1 - c_2^2 c_3^2 - 2c_3\sqrt{(1-c_2^2)(1-c_3^2)}\,\sin\theta} \cdot \exp\left\{-j\,\mathrm{tg}^{-1}\left[\mathrm{tg}\,\theta - \frac{c_3\sqrt{1-c_2^2}}{\sqrt{1-c_3^2}\cos\theta}\right]\right\} \tag{22}$$

where c_1, c_2, and c_3 are the coupling coefficient of coupler C_1, C_2, and C_3, respectively; θ is the phase difference between coupler C_2 and the phase shifter Φ; the effects of

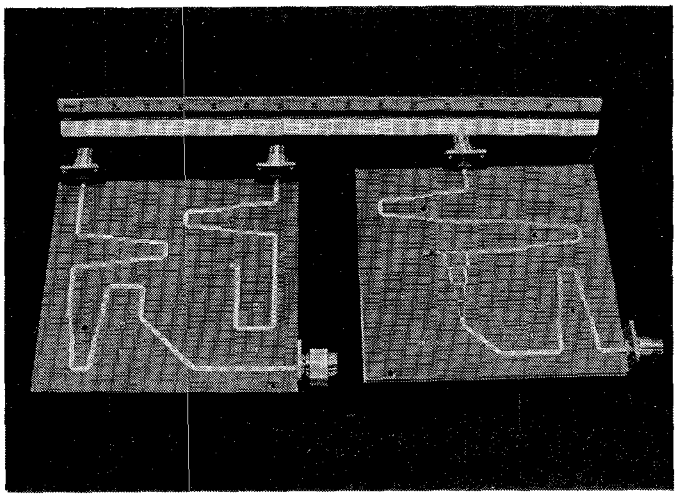

Fig. 5. A photograph of the experimental FPL reflectometer.

residual mismatches of power detectors, adaptors, couplers, power divider, etc., are omitted; and the power divider D is supposed to be precisely 3 dB.

Computer simulation results [8] show that the optimum phase angle ϕ between A_i's is $2\pi/3$ from the point of view of the minimum measurement error. Therefore, an optimized design of an FPL must satisfy the condition of (16), (17), and $\phi = 2\pi/3$ in a frequency band as wide as possible. Obviously, from (18) to (22) one can find that if

$$c_1 = c_3 = 0.707 \text{ (3 dB)} \tag{23}$$

$$c_2 = 0.8165 \text{ (1.76 dB)} \tag{24}$$

and

$$\theta = 0 \tag{25}$$

then an ideal FPL would be achieved. Under these conditions, the expected five-port network parameters will be

$$A_1 = 0.707 \exp(j90°) \tag{26}$$

$$A_2 = 0.707 \exp(-j30°) \tag{27}$$

$$A_3 = 0.707 \exp(-j150°) \tag{28}$$

while q_1 and q_2 are proportional to the sensitivity and gain of related power measurement circuits.

By the use of the documented synthesis technique for symmetrical couplers [9] and power dividers [10], a computer-aided design program was developed by the authors in the FORTRAN-77 language of the Intel-225M microcomputer development system. Using this program, the required input parameters are: expected bandwidth (up to 9:1 octave), and the physical parameters of the microstrip line substrate to be used; then all the circuit's parameters and geometric dimensions of the microstrip line will be calculated and printed out. Fig. 5 shows a photo of the five-port designed by this consideration for the frequency band 2 to 8 GHz. Calibration results (2 to 4 GHz) show that the maximum tolerances obtained are ± 25 percent in amplitude and $\pm 30°$ in phase of the theoretical network parameters found in (26)-(28). These tolerances arise from the mismatches between components and power detectors, especially from the input reflection of the power divider [8].

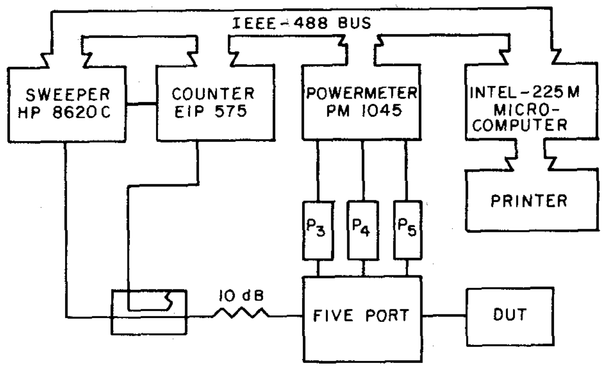

Fig. 6. Block diagram of the experimental automatic FPL reflectometer.

IV. Experiments and Discussion

The block diagram of the experimental five-port reflectometer is shown in Fig. 6, where the FPL is shown in Figs. 4 and 5. All instruments in this system—sweeper (HP8620C with 8621B plug-in), counter (EIP 575), and power meter (PM 1045)—are controlled by micro-computer Intel 225M with IEEE 488 bus. The measured results are printed out in the form of a table and graph.

Before doing any measurement, the five-port reflectometer must be calibrated. The calibration procedure for a five-port has been reported by the authors [11], and the standards used are four short positions of a precision sliding short Model Narda 901NF. The calibration error could be expected within ± 1 percent, including the power measurement error and the reflection coefficient errors of the chosen standards.

Figs. 7 and 8 show some measured reflection coefficients by this reflectometer in the frequency band 2 to 4 GHz (the FRL is designed to be used in a frequency band from 2 to 8 GHz, but only a sweeper from 2 to 4 GHz was available). Fig. 7 shows a group of measurement results for the amplitudes of different reflections, where the DUT are a fixed short, and $|S_{12}S_{21}|$ of four high quality attenuators of 3 dB, 6 dB, 10 dB, and 20 dB. It must be noted that the value of $|S_{12}S_{21}|$ of a two-port network can be measured by a reflectometer using

$$|S_{12}S_{21}| = \frac{1}{2}|\Gamma' - \Gamma''||1 - S_{22}^2| \tag{29}$$

where S_{ij} is the related scattering matrix parameter of the two-port network, and Γ' and Γ'' are the measured complex reflection coefficients when port 2 is opened and shorted, respectively.

The terms S_{22} for every attenuator were measured in June 1981 at the National Bureau of Standards (NBS), Boulder, CO.; all $|S_{22}| < 0.02$ were in the frequency band from 2 to 8 GHz. Therefore the term S_{22}^2 can be omitted in (29) and the resulting error would be less than ± 0.0035 dB.

So, the insertion loss of an attenuator can be measured

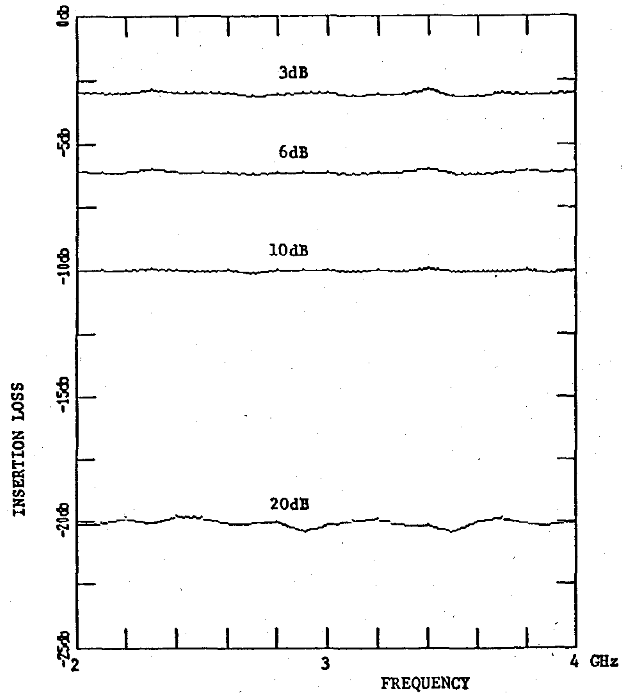

Fig. 7. Measured different reflection coefficient modulus versus frequency.

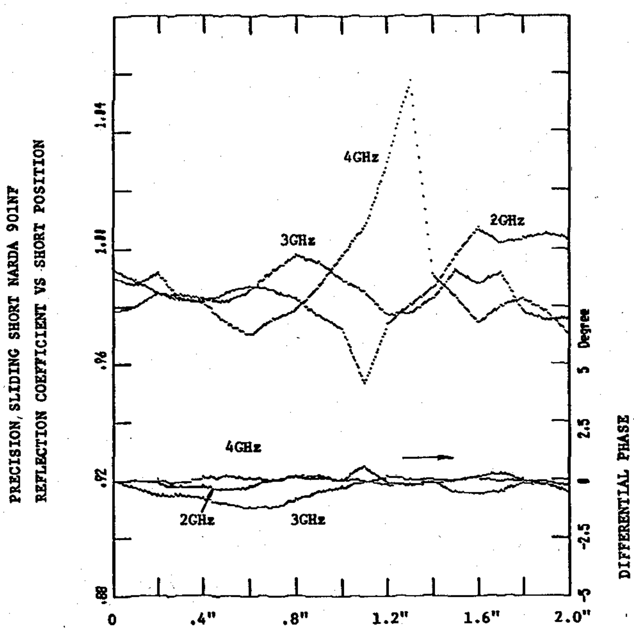

Fig. 8. Measured complex reflection coefficient of a sliding short versus short positions at different frequencies.

as

$$|S_{12}| = 0.7\sqrt{|\Gamma' - \Gamma''|} \tag{30}$$

and the measured results are shown in Fig. 7. A comparison is made with the attenuation values marked by the manufacturer (Midwest) and the values measured in June 1981 at NBS as shown in Table I.

Some measurement results of a coaxial sliding short are reported in Fig. 8 for different frequencies. For more clarity, the phases are reported in the form of phase difference between the measured and the mechanical phase change, which can be calculated from the related displacement of the short position.

TABLE I
COMPARISON OF ATTENUATION MEASUREMENTS DONE ON FOUR COMMERCIAL COMPONENTS

Insertion loss of Attenuators (in dB)*

Midwest Model No.217	Marked value	Measured by FPL Min	Measured by FPL Max	Measured at NBS Min	Measured at NBS Max
3 dB	2.9	2.81	3.05	2.83	3.07
6 dB	6.0	5.93	6.11	5.91	6.13
10 dB	9.9	9.82	10.01	9.84	10.07
20 dB	20.0	19.78	20.37	19.90	20.09

* In the frequency band of 2 to 4 GHz.

It is found that there are relatively large errors in some short positions. It must be noted that the larger error is possible for a FPL when $|\Gamma|$ under test is close to 1 [8].

V. Conclusion

According to the above theoretical analysis, computer simulation, and experimental results, the following conclusions can be made.

a) It is possible and reasonable to use a five-port junction to measure the complex reflection coefficient of a passive load ($|\Gamma| \leqslant 1$). The ambiguity can be avoided and the measurement error is acceptable for most engineering applications.

b) The presented optimized FPL circuit (Fig. 4 and (22) to (25)) provides a way to build a multi-octave five-port reflectometer. Moreover, an optimized six-port circuit can be easily derived from this circuit [8].

References

[1] G. F. Engen, "Determination of microwave phase and amplitude from power measurements," *IEEE Trans. Inst. Measur.*, vol. IM-25, pp. 414–418, Dec. 1976.

[2] G. F. Engen, "The six-port reflectometer: An alternative network analyzer," *IEEE Trans. Microwave Theory Tech.* vol. MTT-25, pp. 1075–1080, 1977.

[3] D. Woods, "Analysis and calibration theory of the general 6-port reflectometer employing four amplitude detectors," *Proc. Inst. Elec. Eng.*, vol. 126, pp. 221–228, 1979.

[4] G. P. Riblet, "A compact waveguide 'resolver' for the accurate measurement of complex reflection and transmission coefficient using the 6-port measurement concept," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 155–162, 1981.

[5] G. P. Riblet, "A broad-band stripline or coaxial 'resolver' for the accurate measurement of complex reflection coefficient using 6-port measurement concept," in *1981 IEEE MTT-S Int. Microwave Symp. Dig.*, Los Angeles, June 1981, pp. 154–156.

[6] S. Li and R. G. Bosisio, "A microcomputer operated five-port reflectometer," *Acta Electron.*, (China), vol. 10, no. 2, pp. 1–6, Mar. 1982.

[7] E. Martin, J. Margineda, and J. M. Zamarro, "An automatic network analyzer using a slotted line reflectometer," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 667–670, May 1982.

[8] S. Li, "Automatic analysis of multi-port microwave networks," Ph.D thesis, Ecole Polytechnique, Univ. of Montreal, 1982.

[9] E. G. Cristal and L. Young, "Theory and tables of optimum symmetrical TEM-mode coupled-transmission-line directional cou-

plers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-13, pp. 544–558, Sept. 1965.

[10] S. B. Cohn, "A class of broad-band three-port TEM-mode hybrids," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 110–116, Feb. 1968.

[11] S. Li and R. G. Bosisio, "Calibration of multi-port reflectometer by means of four short/open circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 1085–1089, July 1982.

Shihe Li (S'81–M'83) was born in Chongqing, Sichuan, China, on April 28, 1941. He graduated from the Chentu Institute of Radio Engineering, China, in 1963, and finished his graduate program at the Department of Physics, Nanking University, China, in 1966. He received the Ph.D. degree from the Ecole Polytechnique, University of Montreal, Montreal, Canada, in 1982.

From 1968 to 1979, he was a Research Engineer at the Fourth Research Institute, Ministry of Posts and Telecommunications, China. He had engaged in the development of high-efficient reflector antennas, microwave ferrite materials, low-loss nonreciprocal microwave devices, and microwave IC's. He arrived at the Ecole Polytechnique de Montreal, Montreal, Canada, in 1980, and did some research and developments on six-port automatic network analyzer techniques. He is now a Member of the Technical Staff in the Fourth Research Institute, Ministry of Posts and Telecommunications, Xian, People's Republic of China.

Dr. Li's interests are in electromagnetic theory and computer-aided microwave design and measurement.

Renato G. Bosisio (M'79) was born in Monza, Italy, on June 28, 1930. He received the B.Sc. degree from McGill University, Montreal, Quebec, Canada, in 1951, and the M.S.E.E. degree from the University of Florida, Gainesville, in 1963.

He has been engaged in microwave research and development since 1957 with various firms in Canada (Marconi and Varian), in the U.S. (Sperry), and in England (English Electric). He is presently Head of the Section d'Electromagnétisme et d'Hyperfréquences at the Ecole Polytechnique de Montréal, University of Montreal, where he teaches microwave theory and techniques and is actively engaged in microwave power applications, instrumentation, and dielectric measurements.

Professor Bosisio is a member of IMPI, Phi Kappa Phi, Sigma Xi, and l'Ordre des Ingénieurs du Québec.

Finite Element Analysis of Lossy Waveguides–Application to Microstrip Lines on Semiconductor Substrate

MICHEL AUBOURG, JEAN-PIERRE VILLOTTE, FRANCK GODON, AND YVES GARAULT

Abstract —**The development of Maxwell's equations is made considering the electromagnetic fields as vector distributions. With the aid of the finite element method, an analysis of lossy shielded inhomogenous waveguides of arbitrary shape is described.**

To solve the complex matrix system an iterative procedure is presented.

The method is applied to study the propagation on MIS or Schottky contact microstrip lines.

I. Introduction

THE FINITE ELEMENT method applied to hybrid wave analysis is commonly used to study propagation along quasi-planar lines like microstrip or coplanar lines [1], [2] and along dielectric-loaded waveguides [3] or dielectric waveguides [4]. In these analyses, materials are considered lossless. It is generally sufficient because substrate-like alumina or semi-insulating GaAs suited for microwave integrated circuits are low-loss media. Hence, skin losses are the most important and can be calculated from the current densities on the conductors.

But in monolithic microwave integrated circuits, to decrease the size of elements it is necessary to have a slow wave propagation medium. It is possible to obtain very slow wave propagation by using MIS or Schottky contact realized on a semiconducting substrate [5]–[7]. In this case, the propagation constant and electromagnetic field are complex.

The purpose of this paper is to present, with the aid of the finite element method, a two-dimensional analysis of the propagation in inhomogeneous lossy waveguides. The development of Maxwell's equation is made considering

Manuscript received December 14, 1981; revised August 2, 1982. This work was supported by the D.A.I.I. under contract 80.35.188.007907500.

The authors are with the Laboratoire d'Electronique des Microondes, Université de Limoges, 87060 Limoges, Cedex, France.